SEM-2 (MATHEMATICS) (GENERAL)
ANUP RAY (06/04/2020)
Differential Equation

Answer (Home work)

1) $x + \frac{y^2}{x^2} = c$ 2) $x^2 - y^2 = cx$

3) $x + y - \frac{y^2+1}{x} = c$ 4) $e^x(x^2 + y^2) = c$

Rule: 3: If $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = f(y)$, a function of y alone, then $e^{\int f(y)dy}$ is an integrating factor of $Mdx + Ndy = 0$

Ex: Solve $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

Ans: Comparing the given equations with the equation $Mdx + Ndy = 0$ we get

$M = 2xy^4e^y + 2xy^3 + y$, $N = x^2y^4e^y - x^2y^2 - 3x$

$\therefore \frac{\partial M}{\partial y} = 8xy^3e^y + 6xy^2 + 2xy^4e^y + 1$

$\frac{\partial N}{\partial x} = 2xy^4e^y - 2xy^2 - 3$

$\therefore \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = -8xy^2 - 8xy^3e^y - 4$

$= -\frac{4}{y}(2xy^4e^y + 2xy^3 + y)$

$\Rightarrow \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = -\frac{4}{y}$, which is a function of y alone.

$\Rightarrow$ I.F. $= e^{-\int \frac{4}{y}dy} = e^{-4\log y} = \frac{1}{y^4}$

Multiplying the given equation by I.F (ie $\frac{1}{y^4}$) we get,

$$\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \left(x^2 e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}\right)dy =$$

which must be exact equation.

$\therefore \int M\,dx$ (Taking y as constant)

$$= e^y \int 2x\,dx + \frac{1}{y}\int 2x\,dx + \frac{1}{y^3}\int dx$$

$$= x^2 e^y + \frac{x^2}{y} + \frac{x}{y^3}$$

$\int$ (Terms in N not containing x) $dy = 0$

$\therefore$ The general solution of the given equation is,

$$x^2 e^y + \frac{x^2}{y} + \frac{x}{y^3} = c$$

Home work

Solve:

1) $(xy^2 - x^2)dx + (3x^2y^2 + x^2y - 2x^3 + y^2)dy = 0$

2) $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

3) $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$

4) $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$